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Inverse scattering problem for a hyperbolic system of first order equations on a semi-axis on a first approximation

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Abstract. *In this paper for a hyperbolic system of five equations on the semi-axis, by joint consideration of three problems an inverse scattering problem on a first approximation was solved. The coefficients of the considered system are uniquely determined by the scattering operator on the semi-axis.*

Keywords. inverse problem, scattering operator, factorization.

Mathematics Subject Classification (2010): 35J25; 35R30, 53C15

1 Introduction

Inverse scattering problems for different linear systems of first order hyperbolic equations on the axis and semi-axis were studied in the papers of L.P. Nizhnik [3], L.P. Nizhnik and V.G. Tarasov [2], A.S. Fokas and L.Y. Sung [1], N.Sh. Iskenderov [4], M.I. Ismailov [5] and others.

In this paper we study direct and inverse scattering problems for a system of five hyperbolic equations of first order on a semi-axis in the case when there are two given incident waves.

When there are three incident and two scattering waves, these problems were studied in [8] when there are four incident and two scattering waves, in [7].

2 Scattering problem on a semi-axis

On the semi-axis $x \geq 0$ consider a system of equations of the form:

$$\xi_i \frac{\partial U_i(x,t)}{\partial t} - \frac{\partial U_i(x,t)}{\partial x} = \sum_{j=1}^{5} c_{ij}(x,t)\, U_j(x,t), i = \overline{1,5} \tag{2.1}$$

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where $c_{ij}(x,t)$ are complex-valued measurable functions with respect to x and t satisfying the conditions:

$$|c_{ij}(x,t)| \leq C\left[(1+|x|)(1+|t|)\right]^{-1-\varepsilon}, \tag{2.2}$$

moreover

$$c_{ii}(x,t)=0, i=\overline{1,5},\ \xi_1>\xi_2>0>\xi_3>\xi_4>\xi_5,\ -\infty<t<+\infty$$

Let us consider system (2.1) on a semi-axis under three different boundary conditions:

$$1) \quad \begin{cases} U_3^1(0,t)=U_1^1(0,t)+U_2^1(0,t) \\ U_4^1(0,t)=U_2^1(0,t) \\ U_5^1(0,t)=U_1^1(0,t) \end{cases} \tag{2.3}$$

$$2) \quad \begin{cases} U_3^2(0,t)=U_1^2(0,t) \\ U_4^2(0,t)=U_1^2(0,t)+U_2^2(0,t) \\ U_5^2(0,t)=U_2^2(0,t) \end{cases} \tag{2.4}$$

$$3) \quad \begin{cases} U_3^3(0,t)=U_2^3(0,t) \\ U_4^3(0,t)=U_1^3(0,t) \\ U_5^3(0,t)=U_1^3(0,t)+U_2^3(0,t) \end{cases} \tag{2.5}$$

Any essentially bounded solution $U(x,t)=\{U_1(x,t),U_2(x,t),...,U_5(x,t)\}$ of the system (2.1) with the coefficients $c_{ij}(x,t), i,j=\overline{1,5}$, satisfying conditions (2.2) admit on the semi-axis $x\geq 0$ the following asymptotic representations:

$$\begin{cases} U_i(x,t)=a_i(t+\xi_i x)+o(1), i=1,2 \\ U_i(x,t)=b_i(t+\xi_i x)+o(1), i=\overline{3,5}, x\to+\infty, \end{cases} \tag{2.6}$$

where $a_i(s) \in L_\infty(-\infty,+\infty)$ $(i=1,2)$ determine the incident waves, while $b_i(s)\in L_\infty(-\infty,+\infty), i=\overline{3,5}$ the scattering ones.

The scattering problem for system (2.1) is in finding the solution to the system (2.1) by the given incident waves and boundary conditions for $x=0$.

The scattering problem under joint consideration of the first, second and third problems is stated as follows: by the given function $a_1(s),a_2(s)\in L_\infty(R), R=(-\infty,+\infty)$ find the solution

$$U^k(x,t)\in L_\infty((0,+\infty)\times(-\infty,+\infty),C^2),(k=1,2)$$

of the first, second and third problems for which in L_∞ the following asymptotic representations are valid:

$$U_i^k(x,t)=a_i(t+\xi_i x)+o(1), x\to\infty, i=1,2, k=\overline{1,3};$$

where $U^k(x,t)=(U_1^k(x,t),...,U_5^k(x,t))$.

Theorem 2.1 *Let the coefficients $c_{ij}(x,t), i,j=5$ of the system* (2.1)*, satisfy conditions* (2.2)*. Then there exists a unique solution of the scattering problem on the semi-axis $x\geq 0$ for the system* (2.1) *with arbitrary given incident waves*

$$a_i(s)\in L_\infty(R), R=(-\infty,+\infty), i=1,2$$

The proof of this theorem is similar to one in [7].

Note that the scattering problem for the k -th $(k = \overline{1,3})$ problem is equivalent to the following system of integral equations:

$$
\begin{aligned}
&U_1^k(x,t) = a_1(t+\xi_1 x) + \int\limits_x^{+\infty} \sum_{j=1}^{5} (c_{1j}U_j)(y,t+\xi_1(x-y))dy,\\
&U_2^k(x,t) = a_2(t+\xi_2 x) + \int\limits_x^{+\infty} \sum_{j=1}^{5} (c_{2j}U_j)(y,t+\xi_2(x-y))dy,\\
&U_i^k(x,t) = b_i(t+\xi_i x) + \int\limits_x^{+\infty} \sum_{j=1}^{5} (c_{ij}U_j)(y,t+\xi_i(x-y))dy, i=\overline{3,5},
\end{aligned}
\tag{2.7}
$$

where the functions $b_3^k(s), b_4^k(s), b_5^k(s) k = 1,2$ are expressed by $a_1(s), a_2(s)$ the coefficients $c_{ij}(x,t), i,j = \overline{1,5}$ and the solutions of the first, second and third problems, respectively, in the following way:

$$
\begin{cases}
b_3^1(t) = a_1(t) + a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j(y,t-\xi_1 y)U_j^1(y,t-\xi_1 y)} + \\
+c_{2j}(y,t-\xi_2 y)U_j^1(y,t-\xi_2 y) - c_{3j}(y,t-\xi_3 y)U_j^1(y,t-\xi_3 y)]dy,\\
b_4^1(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{2j(y,t-\xi_2 y)U_j^1(y,t-\xi_2 y) - c_{4j}(y,t-\xi_4 y)U_j^1(y,t-\xi_4 y)]dy},\\
b_5^1(t) = a_1(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^1(y,t-\xi_1 y) - \\
-c_{5j}(y,t-\xi_5 y)U_j^1(y,t-\xi_5 y)]dy
\end{cases}
\tag{2.8}
$$

$$
\begin{cases}
b_3^2(t) = a_1(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^2(y,t-\xi_1 y) - \\
-c_{3j}(y,t-\xi_3 y)U_j^2(y,t-\xi_3 y)]dy,\\
b_4^2(t) = a_1(t) + a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^2(y,t-\xi_1 y) + \\
+c_{2j}(y,t-\xi_2 y)U_j^2(y,t-\xi_2 y) - c_{4j}(y,t-\xi_4 y)U_j^2(y,t-\xi_4 y)]dy,\\
b_5^2(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{2j}(y,t-\xi_2 y)U_j^2(y,t-\xi_2 y) - \\
-c_{5j}(y,t-\xi_5 y)U_j^2(y,t-\xi_5 y)]dy
\end{cases}
\tag{2.9}
$$

$$
\begin{cases}
b_3^3(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{2j}(y,t-\xi_2 y)U_j^3(y,t-\xi_2 y) - \\
-c_{3j}(y,t-\xi_3 y)U_j^3(y,t-\xi_3 y)]dy,\\
b_4^3(t) = a_1(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^3(y,t-\xi_1 y) - \\
-c_{4j}(y,t-\xi_4 y)U_j^3(y,t-\xi_4 y)]dy\\
b_5^3(t) = a_1(t) + a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^3(y,t-\xi_1 y) + \\
+c_{2j}(y,t-\xi_2 y)U_j^3(y,t-\xi_2 y) - c_{5j}(y,t-\xi_5 y)U_j^3(y,t-\xi_5 y)]dy
\end{cases}
\tag{2.10}
$$

It follows from theorem (2.1) that to each vector-function $a(t) = (a_1(t), a_2(t)) \in L_\infty(R)$ giving the incident waves there correspond the solutions of three scattering problems of the system (2.1) with boundary conditions (2.3), (2.4), (2.5) and the given asymptotics

$$\begin{cases} U_3^k(x,t) = b_3^k(t+\xi_3 x) + o(1), \\ U_4^k(x,t) = b_4^k(t+\xi_4 x) + o(1), \\ U_5^k(x,t) = b_5^k(t+\xi_5 x) + o(1), k = \overline{1,3}, x \to +\infty, \end{cases} \tag{2.11}$$

i.e. the vector of scattering waves $b(t) = \left(b^1(t), b^2(t), b^3(t)\right)$,where $b^k(t) = \left(b_3^k(t), b_4^k(t), b_5^k(t)\right) \left(k = \overline{1,3}\right)$. Relation (2.11) follows from (2.7) and conditions (2.2). Thus, in the space of essentially bounded functions, we determined the operator $S = \left(S^1, S^2, S^3\right)$ that takes $a(t)$ to $b(t)$:

$$S^k \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = \begin{pmatrix} b_3^k(t) \\ b_4^k(t) \\ b_5^k(t) \end{pmatrix}, k = \overline{1,3} \tag{2.12}$$

Here,

$$S = \left(S^1, S^2, S^3\right) \text{ and } S^k = \begin{pmatrix} S_{11}^k S_{12}^k \\ S_{21}^k S_{22}^k \\ S_{31}^k S_{32}^k \end{pmatrix}, k = \overline{1,3}$$

$$\begin{array}{l} b_3^k(t) = S_{11}^k a_1(t) + S_{12}^k a_2(t), \\ b_4^k(t) = S_{21}^k a_1(t) + S_{22}^k a_2(t), \\ b_5^k(t) = S_{31}^k a_1(t) + S_{32}^k a_2(t), k = \overline{1,3.} \end{array} \tag{2.13}$$

From (2.8), (2.9), (2.10) it follows that the elements of the operator have $S^k(k = \overline{1,3})$ have the form:

$$\begin{cases} S_{11}^1 = I + F_{11}^1, S_{12}^1 = I + F_{12}^1, \\ S_{21}^1 = F_{21}^1, S_{22}^1 = I + F_{22}^1, \\ S_{31}^1 = I + F_{31}^1, S_{32}^1 = F_{32}^1, \end{cases}$$

$$\begin{cases} S_{11}^2 = I + F_{11}^2, S_{12}^2 = F_{12}^2, \\ S_{21}^2 = I + F_{21}^2, S_{22}^2 = I + F_{22}^2, \\ S_{31}^2 = F_{32}^2, S_{32}^2 = I + F_{32}^2, \end{cases} \tag{2.14}$$

$$\begin{cases} S_{11}^3 = F_{11}^3, S_{12}^3 = I + F_{12}^3, \\ S_{21}^3 = I + F_{21}^3, S_{22}^3 = F_{22}^3, \\ S_{31}^3 = I + F_{31}^3, S_{32}^3 = I + F_{32}^3, \end{cases}$$

where the operators $F_{ij}^k(j = 1, 2,\ i, k = \overline{1,3,})$ are Fredholm integral operators.

3 The inverse scattering problem on a semi-axis on a first approximation

The inverse problem for the system (2.1), is in finding the coefficients of the system (2.1), by the given scattering operator S on a semi-axis.

Here the coefficients of the system (2.1), are restored by the scattering operator on a semi-axis constructed on a first approximation. It is constructed in the explicit form.

For the first problem as a zero order approximation we take

$$\begin{aligned}
&U_k^{(1,0)}(x,t) = a_k(t+\xi_k x), k=1,2.\\
&U_3^{(1,0)}(x,t) = a_1(t+\xi_3 x) + a_2(t+\xi_3 x),\\
&U_4^{(1,0)}(x,t) = a_2(t+\xi_4 x),\\
&U_5^{(1,0)}(x,t) = a_1(t+\xi_5 x);
\end{aligned}$$

for the second problem

$$\begin{aligned}
&U_k^{(2,0)}(x,t) = a_k(t+\xi_k x), k=1,2.\\
&U_3^{(2,0)}(x,t) = a_1(t+\xi_3 x),\\
&U_4^{(2,0)}(x,t) = a_1(t+\xi_4 x) + a_2(t+\xi_4 x),\\
&U_5^{(2,0)}(x,t) = a_2(t+\xi_5 x);
\end{aligned}$$

for the third problem

$$\begin{aligned}
&U_k^{(3,0)}(x,t) = a_k(t+\xi_k x), k=1,2.\\
&U_3^{(3,0)}(x,t) = a_2(t+\xi_3 x),\\
&U_4^{(3,0)}(x,t) = a_1(t+\xi_4 x),\\
&U_5^{(3,0)}(x,t) = a_1(t+\xi_5 x) + a_2(t+\xi_5 x);
\end{aligned}$$

Then in equalities (2.8), (2.9)and (2.10) the first order approximations will be:

$$\begin{cases}
b_3^1(t) = a_1(t) + a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^{(1,0)}(y,t-\xi_1 y) +\\
+c_{2j}(y,t-\xi_2 y)U_j^{(1,0)}(y,t-\xi_2 y) - c_{3j}(y,t-\xi_3 y)U_j^{(1,0)}(y,t-\xi_3 y)]dy,\\
b_4^1(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{2j}(y,t-\xi_2 y)U_j^{(1,0)}(y,t-\xi_2 y) -\\
-c_{4j}(y,t-\xi_4 y)U_j^{(1,0)}(y,t-\xi_4 y)]dy,\\
b_5^1(t) = a_1(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^{(1,0)}(y,t-\xi_1 y) -\\
-c_{5j}(y,t-\xi_5 y)U_j^{(1,0)}(y,t-\xi_5 y)]dy;
\end{cases} \tag{3.1}$$

$$\begin{cases}
b_3^2(t) = a_1(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^{(2,0)}(y,t-\xi_1 y) -\\
-c_{3j}(y,t-\xi_3 y)U_j^{(2,0)}(y,t-\xi_3 y)]dy,\\
b_4^2(t) = a_1(t) + a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y,t-\xi_1 y)U_j^{(2,0)}(y,t-\xi_1 y) +\\
+c_{2j}(y,t-\xi_2 y)U_j^{(2,0)}(y,t-\xi_2 y) -\\
-c_{4j}(y,t-\xi_4 y)U_j^{(2,0)}(y,t-\xi_4 y)]dy,\\
b_5^2(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{2j}(y,t-\xi_2 y)U_j^{(2,0)}(y,t-\xi_2 y) -\\
-c_{5j}(y,t-\xi_5 y)U_j^{(2,0)}(y,t-\xi_5 y)]dy;
\end{cases} \tag{3.2}$$

$$
\begin{cases}
b_3^3(t) = a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{2j}(y, t-\xi_2 y) U_j^{(3,0)}(y, t-\xi_2 y) - \\
-c_{3j}(y, t-\xi_3 y) U_j^{(3,0)}(y, t-\xi_3 y)] dy, \\
b_4^3(t) = a_1(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y, t-\xi_1 y) U_j^{(3,0)}(y, t-\xi_1 y) - \\
-c_{4j}(y, t-\xi_4 y) U_j^{(3,0)}(y, t-\xi_4 y)] dy, \\
b_5^3(t) = a_1(t) + a_2(t) + \int\limits_0^{+\infty} \sum\limits_{j=1}^{5} [c_{1j}(y, t-\xi_1 y) U_j^{(3,0)}(y, t-\xi_1 y) + \\
+c_{2j}(y, t-\xi_2 y) U_j^{(3,0)}(y, t-\xi_2 y) - c_{5j}(y, t-\xi_5 y) U_j^{(3,0)}(y, t-\xi_5 y)] dy;
\end{cases}
\tag{3.3}
$$

respectively.

Taking into account in (2.13) and (2.14), we have:

$$c_{12}(x,y) = (\xi_2 - \xi_1)\left[F_{12}^1(\xi_1 x + y, \xi_2 x + y) - F_{12}^2(\xi_1 x + y, \xi_2 x + y)\right],$$

$$c_{13}(x,y) = (\xi_1 - \xi_3)\Big[F_{12}^2(\xi_1 x + y, \xi_3 x + y) - F_{12}^1(\xi_1 x + y, \xi_3 x + y) +$$

$$+\frac{1}{2}[F_{22}^3(\xi_1 x + y, \xi_3 x + y) + F_{32}^1(\xi_1 x + y, \xi_3 x + y) - F_{21}^3(\xi_1 x + y, \xi_3 x + y)]\Big]$$

$$c_{14}(x,y) = \frac{\xi_1 - \xi_4}{2}\Big[F_{21}^3(\xi_1 x + y, \xi_4 x + y) - F_{22}^3(\xi_1 x + y, \xi_4 x + y)$$

$$+F_{32}^1(\xi_1 x + y, \xi_4 x + y)\Big],$$

$$c_{15}(x,y) = (\xi_1 - \xi_5)[F_{31}^1(\xi_1 x + y, \xi_5 x + y) - F_{12}^2(\xi_3 x + y, \xi_5 x + y) + F_{12}^{1`}(\xi_3 x + y, \xi_5 x + y) +$$

$$+\frac{1}{2}[F_{21}^3(\xi_1 x + y, \xi_5 x + y) - F_{22}^3(\xi_1 x + y, \xi_5 x + y) - F_{32}^1(\xi_1 x + y, \xi_5 x + y)]],$$

$$c_{21}(x,y) = (\xi_1 - \xi_2)\left[F_{11}^1(\xi_2 x + y, \xi_1 x + y) - F_{11}^2(\xi_2 x + y, \xi_1 x + y)\right],$$

$$c_{23}(x,y) = (\xi_2 - \xi_3)\Big[F_{22}^1(\xi_2 x + y, \xi_3 x + y) - F_{11}^2(\xi_2 x + y, \xi_3 x + y) +$$

$$+ F_{12}^1(\xi_2 x + y, \xi_3 x + y)\Big],$$

$$c_{24}(x,y) = (\xi_4 - \xi_2)\left[F_{11}^2(\xi_2 x + y, \xi_4 x + y) - F_{12}^1(\xi_2 x + y, \xi_4 x + y)\right],$$

$$c_{25}(x,y) = (\xi_2 - \xi_5)[F_{32}^2(\xi_2 x + y, \xi_5 x + y) - F_{11}^2(\xi_2 x + y, \xi_5 x + y) +$$

$$+F_{12}^1(\xi_2 x + y, \xi_5 x + y)],$$

$$c_{31}(x,y) = (\xi_3 - \xi_1)F_{11}^2(\xi_3 x + y, \xi_1 x + y),$$

$$c_{32}(x,y) = (\xi_3 - \xi_2)F_{12}^2(\xi_3 x + y, \xi_2 x + y),$$

$$c_{34}(x,y) = (\xi_4 - \xi_3)[F_{11}^3(\xi_3 x + y, \xi_4 x + y) - F_{12}^3(\xi_3 x + y, \xi_4 x + y) + 2F_{12}^1(\xi_3 x + y, \xi_4 x + y) -$$

$$-2F_{11}^2(\xi_3 x + y, \xi_4 x + y) - F_{22}^1(\xi_3 x + y, \xi_4 x + y)],$$

$$c_{35}(x,y) = (\xi_5 - \xi_3)[F_{12}^3(\xi_3 x + y, \xi_5 x + y) - 2F_{12}^1(\xi_3 x + y, \xi_5 x + y) + 2F_{11}^2(\xi_3 x + y, \xi_5 x + y) -$$

$$-F_{32}^2(\xi_3 x + y, \xi_5 x + y) - F_{22}^1(\xi_3 x + y, \xi_5 x + y)],$$

$$c_{41}(x,y) = (\xi_4 - \xi_1)F_{21}^3(\xi_4 x + y, \xi_1 x + y),$$

$$c_{42}(x,y) = (\xi_4 - \xi_2)F^2_{22}(\xi_4 x + y, \xi_2 x + y),$$
$$c_{43}(x,y) = (\xi_4 - \xi_3)\left[F^1_{22}(\xi_4 x + y, \xi_3 x + y) - F^2_{22}(\xi_4 x + y, \xi_3 x + y)\right],$$
$$c_{45}(x,y) = (\xi_5 - \xi_4)[F^2_{22}(\xi_4 x+y, \xi_5 x+y) - F^3_{31}(\xi_4 x+y, \xi_5 x+y) - F^1_{12}(\xi_4 x+y, \xi_5 x+y) + \\ + F^2_{12}(\xi_4 x + y, \xi_5 x + y)],$$
$$c_{51}(x,y) = (\xi_5 - \xi_1)[F^1_{31}(\xi_5 x+y, \xi_1 x+y) + F^2_{32}(\xi_5 x+y, \xi_1 x+y) - F^1_{32}(\xi_5 x+y, \xi_1 x+y)],$$
$$c_{52}(x,y) = (\xi_5 - \xi_2)[F^3_{32}(\xi_5 x+y, \xi_2 x+y) - F^1_{32}(\xi_5 x+y, \xi_2 x+y) + F^2_{32}(\xi_5 x+y, \xi_2 x+y)],$$
$$c_{53}(x,y) = (\xi_5 - \xi_3)[F^1_{32}(\xi_5 x + y, \xi_3 x + y) - F^2_{32}(\xi_5 x + y, \xi_3 x + y)],$$
$$c_{54}(x,y) = (\xi_5 - \xi_4)[F^1_{32}(\xi_5 x + y, \xi_4 x + y) - F^3_{32}(\xi_5 x + y, \xi_4 x + y)]. \tag{3.4}$$

where $F^k_{ij}(t,\tau)$ -kernel of the operators $F^k_{ij}(j = 1, 2, k = \overline{1,3})$. Note that from the remaining relations we have:

$$F^1_{21}(t,\tau) = F^2_{21}(t,\tau), \tau > t,$$

$$F^1_{11}(t,\tau) = F^1_{31}(t,\tau) + F^3_{12}(t,\tau), \tau < t,$$

$$F^1_{11}(t,\tau) = F^3_{12}(t,\tau) + F^2_{12}(t,\tau) - F^1_{12}(t,\tau) + F^1_{31}(t,\tau), \tau < t,$$

$$F^1_{12}(t,\tau) = F^3_{11}(t,\tau) - F^3_{12}(t,\tau) - F^2_{11}(t,\tau) - F^1_{22}(t,\tau) + \\ + F^2_{12}(t,\tau) + F^1_{32}(t,\tau), \tau < t,$$
$$F^1_{21}(t,\tau) = F^3_{22}(t,\tau) - F^3_{31}(t,\tau) + F^2_{12}(t,\tau) + F^1_{22}(t,\tau) - \\ -2F^2_{11}(t,\tau) + F^2_{32}(t,\tau) + F^1_{12}(t,\tau), \tau < t,$$

$$\begin{aligned}
&F^2_{11}(t,\tau) = F^3_{11}(t,\tau) - F^3_{12}(t,\tau) + F^1_{12}(t,\tau) - 2F^2_{11}(t,\tau) - \\
&-F^1_{22}(t,\tau) + F^2_{12}(t,\tau) + F^1_{32}(t,\tau), \tau < t, \\
&F^2_{12}(t,\tau) = F^3_{11}(t,\tau) - 2F^1_{22}(t,\tau) - F^2_{32}(t,\tau) + 2F^1_{12}(t,\tau) - 2F^2_{12}(t,\tau) + \\
&+F^3_{21}(t,\tau) - F^3_{22}(t,\tau) + F^1_{31}(t,\tau), \tau < t, \\
&F^2_{21}(t,\tau) = F^2_{12}(t,\tau) + F^1_{32}(t,\tau) - F^1_{12}(t,\tau) + F^1_{22}(t,\tau), \tau < t, \\
&F^2_{21}(t,\tau) = F^3_{21}(t,\tau) + F^1_{22}(t,\tau) - F^2_{22}(t,\tau) + F^1_{11}(t,\tau) - F^2_{11}(t,\tau), \tau > t \\
&F^3_{31}(t,\tau) = F^3_{21}(t,\tau) - F^3_{22}(t,\tau) + F^1_{31}(t,\tau) - F^2_{12}(t,\tau) - \\
&-F^1_{12}(t,\tau) + F^2_{32}(t,\tau), \tau < t, \\
&F^2_{31}(t,\tau) = F^1_{31}(t,\tau) + F^1_{32}(t,\tau) - F^3_{32}(t,\tau) + F^1_{11}(t,\tau) - F^2_{11}(t,\tau), \tau > t, \\
&F^3_{11}(t,\tau) = F^1_{11}(t,\tau), \tau > t, \\
&F^3_{12}(t,\tau) = F^2_{12}(t,\tau), \tau < t \\
&F^3_{22}(t,\tau) = F^1_{22}(t,\tau), \tau > t, \\
&F^3_{31}(t,\tau) = F^1_{31}(t,\tau) + F^2_{32}(t,\tau) - F^3_{32}(t,\tau) + F^1_{11}(t,\tau) - F^2_{11}(t,\tau), \tau > t \\
&F^3_{32}(t,\tau) = F^1_{12}(t,\tau) + F^1_{31}(t,\tau) - F^2_{12}(t,\tau), \tau < t,
\end{aligned} \tag{3.5}$$

The scattering operator S has 18 elements on the axis or 36 elements on the semi-axis. From 36 elements by formula (3.4) we find 20 coefficients of the system (2.1), 16 unnecessary elements are connected with 8 relations on the axis (16 on the semi-axis with respect to t) by formula (3.5).

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