Khazar Journal of Humanities and Social Sciences Volume 22 № 2 2019, 5-19

DOI: 10.5782/2223-2621.2019.22.2.5

'And' is not always a logical conjunction

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Abstract

The mental models theory has shown that the logical connectives do not always refer to the interpretation assigned to them by standard logic. Several papers authored by its proponents clearly reveal that in the cases of the conditional and disjunction. In this paper, following a methodology of analysis akin to that of the mental models theory, I try to check whether or not the same applies to conjunction, and my conclusion is that, indeed, this last connective can be linked to any of the sixteen possible interpretations that a logical operator relating two clauses can have.

Keywords: and; conjunction; logical connective; mental models; semantics

Introduction

A current cognitive theory, the mental models theory (from now on, MMT), has shown that the traditional logical connectives are not necessarily linked to the interpretations that classical logic often attributes to them (see, e.g., Johnson-Laird & Byrne, 2002, for the case of the conditional, and Orenes & Johnson-Laird, 2012, for the case of disjunction). Indeed, as it is well known, the truth tables of standard logic provide clear interpretations for such connectives. However, MMT proposes that there are certain pragmatic and semantic 'modulation' processes (see, e.g., Johnson-Laird, Khemlani, & Goodwin, 2015) that cause those interpretations to be unsuitable.

Many examples of this are to be found in the literature on MMT, and the case of disjunction is very representative. Standard logic provides three possible scenarios in which an inclusive disjunction such as p ∨ q (where '∨', obviously, expresses inclusive disjunctive relationship) can be true: p & q, p & ¬q (where '¬' is the logical negation), and ¬p & q. Certainly, an inclusive disjunction can be correct in three situations: when the two disjuncts are true (p & q), when only the first one is true (p & ¬q), and when only the second one is so (¬p & q). Nevertheless, the disjunctions in natural language do not always admit these three possibilities. Let us think about, for instance, this sentence:

"...Paco visited Paris or he visited France" (Orenes & Johnson-Laird, 2012, p. 375).

Actually, this sentence can only be true in two of the three scenarios previously indicated: p & q and ¬p & q. The situation p & ¬q is, as argued by Orenes and Johnson-Laird (2012), eliminated by modulation, and the reason is evident. If 'p' stands for 'Paco visited Paris' and 'q' represents 'Paco visited France', it cannot be thought that Paco visited Paris and he did not visit France (p & ¬ q), since Paris is the capital of France.

Something similar happens in the case of the conditional. According to classical logic, although they are not exactly the same as those of disjunction, there are also three possible combinations in which a sentence such as p → q (where '→' represents conditional relationship) can be true: p & q, ¬p & q, and ¬p & ¬q. Thus, the situations are now: when both the first and the second clauses are true, when the first clause is false and the second one is true, and when both of them are false. Nonetheless, the problem is the same again. We can find conditionals in natural language that do not refer to the mentioned possibilities, for example,

"If oxygen is present then there may be a fire" (Johnson-Laird & Byrne, 2002, p. 663).

It is obvious that the suitable interpretation for this conditional is not that just indicated, but the one that Johnson-Laird and Byrne call 'Enabling' and that consists of these possibilities: p & q, p & ¬q, and ¬p & ¬q. As it can be noted, it can be thought that Enabling is an inverse conditional, since its truth values match those of a sentence such as q → p (notice, in the same way, that, apart from p & q and ¬p & ¬q, which, as shown below, are the combinations of the biconditional, Enabling is true in the case in which the conditional is false, p & ¬q, and false in the case in which the conditional is true, ¬p & q, and, evidently, vice versa). However, what is important here is that, indeed, the correct interpretation of the last sentence mentioned above is that of Enabling or an inverse conditional, as, if 'p' is 'oxygen is present' and 'q' denotes 'there may be a fire', it is only possible that oxygen is present and there is a fire (p & q), oxygen is present and there is not a fire (p & ¬q), and oxygen is not present and there is not a fire (¬p & ¬q). What cannot occur is that oxygen is not present and there is a fire (¬p & q).

Undoubtedly, this is a very relevant issue, since, for example, Johnson-Laird and Byrne (2002) identified up to ten different possible interpretations for the conditional, which means that, while in many cases the logical connectives can be understood based on the combinations given to them in the truth tables of standard logic (and this is a point that even MMT seems to accept; see, e.g., Johnson-Laird, 2012), this is not always so. Thus, in this paper, I will try to review, following the analysis of possibilities methodology of MMT described, another connective that, as far as I know, has not been studied to the same extent as the conditional and disjunction yet. That connective is conjunction, that is, the one that links two clauses by means of ‘and’ and that is often represented in logic as ‘∧’, and my main aim is to show that the interpretations corresponding to it are all of the possible combinations of clauses sets that can be assigned to a logical operator relating two clauses, that is, sixteen.

Certainly, if we take descriptions such as that of Deaño (1999, p. 89) into account, we can say that, from the combinations p & q, p & ¬q, ¬p & q, and ¬p & ¬q, the combinations sets that can be built are sixteen, merely some examples being that traditionally attributed to the conditional, which, as said, is {p & q, ¬p & q, ¬p & ¬q}, the one usually assigned to the inclusive disjunction, which, as also pointed out, is {p & q, p & ¬q, ¬p & q}, or that generally linked to conjunction, which is {p & q}. Accordingly, given that my basic goal is, as indicated too, to argue that it is possible to refer to all of those sets by means of natural language sentences with conjunctions, I will analyze in turn each of such sets and propose examples of conjunctive sentences in natural language for them. I begin with the one that can be considered to be the simplest in this study: the set that habitually is linked to conjunction.

Conjunction

Really, it is almost trivial to show that a conjunction can be interpreted under the combinations set that is attributed to it in classical logic. As it is well known, and has already been said, this is the usual interpretation of sentences with a structure such as ‘p and q’ (obviously, ‘and’ is the word in English; in other languages it is different), which are often represented as p ∧ q and are related to the set {p & q}. An easy example can be as follows:

This is a car and that is a bicycle

Indeed, using a procedure akin to that why MMT describes possible scenarios (see, e.g., Oakhill & Garnham, 1996), it can be stated that a sentence such as this one can be true in a scenario such as the following:

Car	Bicycle

That is, a scenario in which we can find both a car and a bicycle, or, if preferred, a scenario matching the set {p & q}.

However, a conjunction with two positive conjuncts can also express a deep conjunction in which the second conjunct is denied. This fact leads to the second set.

Conjunction with the second conjunct negated

Now the set is {p & ¬q} but the point is that that set can be the interpretation, as indicated, of a conjunction with no negation in its conjuncts. To find an example in this regard it is only necessary not to forget that many times people speak in a figurative way. Thus, it is not uncommon to hear sentences similar to this one:

You are a very bad person and, for this reason, people love you

Depending on the context and the circumstances, what this sentence can be expressing is that people do not love you (it is said that they do ironically) because you are bad. So, in certain context and circumstances, the only possible scenario would be:

You are a bad person People do not love you

Nevertheless, it can also be the case that the denied conjunct is the first one.

Conjunction with the first conjunct negated

The set is here just the inverse of the previous one, that is, {¬p & q}, and Johnson-Laird and Byrne's (2002) study on conditionals can help us give an example. As mentioned, Johnson-Laird and Byrne (2002) proposed that, among the sixteen possible interpretations, ten of them could be attributed to the conditional. One of them was, in their view, for example, the first one that has been reviewed in this paper, that is, the one corresponding to the logical conjunction, i.e., to the set {p & q}. Nevertheless, because they were analyzing conditionals, they called the sets using names different from those that I am using and those that I will use below. In this way, they resorted to names more related to conditional relationships and called, following with the same example, 'Ponens' to the set {p & q}.

Nonetheless, what is important for this section is that {¬p & q} is also a set of those that they linked to the conditional. Its name in their paper was 'Deny antecedent and affirm consequent' and one of the examples offered was:

"If Bill Gates needs money then I'll lend it to him" (Johnson-Laird & Byrne, 2002, p. 663).

This example is relevant because, from it, it is possible to build a conjunction related to {¬p & q} as well. That conjunction is the following:

Bill Gates needs money and I will lend it to him

It is absolutely clear that this last sentence, as well as the previous conditional, should not be understood literally, since it does not mean that Bill Gates needs money and that I will lend it to him, but that, although Bill Gates does not need money, I will give it to him. So, the real scenario to which the sentence refers is as follows:

Bill Gates does not need money I give money to him

But it is also possible to interpret that a conjunction leads to a deep structure in which the two conjuncts are denied. Johnson-Laird and Byrne’s (2002) work can also be helpful to show that.

Conjunction with the two conjuncts negated

Certainly, Johnson-Laird and Byrne also considered the set {¬p & ¬q} for conditionals. The name given by them is ‘Tollens’ and one of their examples:

“If it works then I’ll eat my hat” (Johnson-Laird & Byrne, 2002, p. 663).

And, again, a conjunction can be thought from this example:

It works and I will eat my hat

In the two cases, the speaker means that it cannot work and that hence he/she will not eat his/her hat. Thus, the only possible scenario is this:

It does not work I do not eat my hat

And this is so, obviously, because what these sentences really express is, as indicated, that it is absolutely impossible that it works, the reference to the hat being only a way to emphasize that idea.

These four interpretations reviewed (conjunction, conjunction with the second conjunct negated, conjunction with the first conjunct negated, and conjunction with the two conjuncts negated) are the interpretations including only one combination. In the next sections, I will address those corresponding to sets with more combinations of possibilities.

P

Another set can be {p & q, p & ¬q}. Johnson-Laird and Byrne (2002) think that this one can be an interpretation of the conditional too, and the name they give it is 'Strengthen antecedent'. However, I prefer to call it just 'P' for at lest two reasons: as in other cases, Johnson-Laird and Byrne's name appears to be suitable only for the conditional (it includes the word 'antecedent'), and 'P' is an appropriate name because, in the two scenarios enabled in this set, p is true, which means that, under this interpretation, p is necessarily true.

In any case, it is very easy to find examples of conjunctions that can be understood by virtue of this set. One of them can be clearly:

There is a house and there may be a river

There is no doubt that, in this example, the scenarios are these:

House	River
House	Not river

True, the first conjunct expresses that there is a house for sure, but the second one, because the word 'may' is used, only refers to a possibility. So, the river may be and may not be.

Likewise, it is not hard to imagine an example in which 'may' is included in the first conjunct, which leads us to the following interpretation.

Q

Indeed, in the case that that verb is in the first conjunct, the sentence cannot be linked to the previous set, but to {p & q, ¬p & q}, which, according to Johnson-Laird and Byrne (2002), represents a possible interpretation of the conditional as well. The name used by them is 'Relevance', but, as above, I name it 'Q' because this clause is always true in all of the scenarios allowed by this set. As pointed out, to think about examples is not difficult in this case either. It can be enough, for instance, simply to change the position of 'may' in the last sentence:

There may be a house and there is a river

Now, what is not known for sure and, therefore, only possible is the presence of the house. On the other hand, the speaker is absolutely sure there is a river. So, the possibilities are:

House	River
Not house	River

Of course, the use of ‘may’ can lead one to take the operator of possibility of modal logic (see, e.g., Blackburn, de Rijke, & Venema, 2010) into account in the cases of P and Q, and in the one of some other interpretation below. Nonetheless, analyzing this operator and its relationships to language is beyond the scope of this paper. As indicated, what is interesting here is to show that conjunction can refer to any combinations of possibilities between two clauses set, and, from this perspective, reviewing the particular words included in the conjuncts is not an essential task. Hence, as far as that aim is concerned, what has been argued so far can be considered to be enough.

Biconditional

It can be said that the biconditional interpretation of the conditional is habitual and natural for some sentences expressed with the words ‘if… then…’ Classical logic provides a truth table for formulae such as $p \leftrightarrow q$ (where ‘$\leftrightarrow$’ is, evidently, the symbol of the biconditional) that reveals that they can only be true in the cases of the combinations set $\{p \ \& \ q, \neg p \ \& \ \neg q\}$, but there is an extensive literature on the pragmatic phenomenon of the conditional perfection, that is, the phenomenon why a sentence such as $p \rightarrow q$ can be understood as $p \leftrightarrow q$, just a few works addressing it being, for example, Auwera (1997), Horn (2000), or Moldovan (2009). MMT does not ignore this phenomenon either. In fact, Johnson-Laird and Byrne (2002) also propose that one of the ten interpretations of the conditional is the biconditional one. However, what I want to highlight here is that a conjunction such as $p \wedge q$ can be actually a biconditional such as $p \leftrightarrow q$ too.

Really, it is not difficult to do that, since there are many sentences with conjunctions expressing biconditional relationships. The following is clearly one of them:

The temperature is below zero and water freezes

There is no doubt that this sentence only enables two possibilities:

Below zero	Water frozen
Not below zero	Not water frozen

And this is so because it is possible neither that the temperature is below zero and water does not freeze nor that the temperature is not below zero and water frezees. So, obviously, conjunctions can also be interpreted biconditionally.

¬Q

In the same way, a conjunction can transmit that the second conjunct is false (i.e., that q is false, or, if preferred, that ¬q is true) in all of the possible scenarios. And the reason of this is that some conjunctions only allow the combinations {p & ¬q, ¬p & ¬q}. To note it, it is only necessary to remember again that people resort to figurative language many times. Let us think about this sentence:

It is possible that I do that and this planet stops

If somebody states something like that, it is clear that he/she does not mean that this planet will stop if he/she does that. What he/she means is that he/she is going to try to do something very difficult and that it is possible that he/she does it. Thus, the second conjunct indicating, figuratively, that the planet will stop only reveals that the activity or task to do is very hard, and that to do it will be quite an achievement. So, the only possibilities are:

I do that	This planet does not stop
I do not do that	This planet does not stop

And the cause is, as said, that, whether or not the speaker does the activity or task, this planet will not stop.

Exclusive disjunction

The exclusive disjunction is other connective of classical logic, and its truth table points out that the cases in which it can be true are just those of the set {p & ¬q, ¬p & q}. The habitual symbol of this kind of disjunction is '$\veebar$', and hence what I wish to show in this section is that some sentences with the structure p $\wedge$ q truly express p $\veebar$ q. A simple example is once again enough to check that. Liu and Chou used this exclusive disjunction in their study:

"It is now daytime or nighttime" (Liu & Chou, 2012, p. 690).

And, based on it, it is not complicated to build a conjunctive sentence with a similar sense:

It may be daytime and it may be nighttime

The verb 'may' is used here again, but what is important is that both of the two last sentences are related to only these scenarios:

Daytime	Not nighttime

Not daytime	Nighttime

Indeed, more scenarios are not possible, as it cannot be daytime and nighttime, and, in the same way, it can be neither daytime nor nighttime.

Furthermore, as an illustration, it can be added that the proponents of MMT has also analyzed disjunction, its habitual sense, and whether its primary interpretation is the inclusive one or the exclusive one in different works. Some examples are Khemlani, Orenes, & Johnson-Laird (2014) and Orenes and Johnson-Laird (2012).

¬P

This is a case akin to that of ¬Q in which the order of the conjuncts is the inverse. Now, the first conjunct is the one that must always be false, the second one having the possibility to be both true and false. So, its combinations of possibilities set is {¬p & q, ¬p & ¬q}, and I give it the name ‘¬P’ because, in a similar way as ¬Q, ¬p is necessarily true under this interpretation.

An easy example of ¬P can be one based on that about Bill Gates in the Conjunction with the first conjunct negated section. The only change it is required to do is to introduce a possibility in the second conjunct in this way:

Bill Gates needs money and it is possible that I lend it to him

Given that, as commented on, the speaker and the listener know that Bill Gates does not really need money, what this sentence indicates is that the possibility exists that the speaker spends money on a Bill Gates’ product. Thus, the possible scenarios are the following:

Bill Gates does not need money	I give money to him
Bill Gates does not need money	I do not give money to him

Nevertheless, a conjunction can refer to sets with three combinations too. The next sections show this.

Inclusive disjunction

As explained, the inclusive disjunction is generally represented in standard logic as $p \vee q$ and its truth table is linked to the set {p & q, p & ¬q, ¬p & q}. The example for this interpretation, which is also assigned to the conditional by Johnson-Laird and Byrne (2002) with the name of ‘Disabling’, can be elaborated from an instance of conditional sentence included in this last paper too:

“If the workers settle for lower wages then the company may still go bankrupt” (Johnson-Laird & Byrne, 2002, p. 663).

Certainly, the construction of a sentence with these two clauses and a conjunction is not difficult:

The workers settle for lower wages and the company may still go bankrupt

And there is no doubt that both sentences are correct in the same cases:

Lower wages	Bankrupt
Lower wages	Not bankrupt
Not lower wages	Bankrupt

That is, clearly, in the possibilities of disjunction when inclusive, a connective to which, as indicated above, MMT has paid attention.

Inverse conditional

It has already been accounted for that, following Johnson-Laird and Byrne, a conditional can be interpreted by means of the set {p & q, p & ¬q, and ¬p & ¬q}, and that they call that interpretation ‘Enabling’. It has also been argued that that very interpretation corresponds to the one of an inverse conditional, and an example (that of oxygen and fire) of sentence with the structure $p \rightarrow q$ valid for those combinations of possibilities coming from Johnson-Laird and Byrne (2002) has been offered as well.

Hence what is necessary now is to give an example of conjunction referring to that same set. To do that is not hard either, since Johnson-Laird and Byrne’s paper can be taken into account again and the example can be based on the mentioned conditional sentence above. In this way, the instance can be as follows:

Oxygen is present and there may be a fire

Obviously, the possibilities of this sentence are the same as those of the previous example with a conditional, that is,

Oxygen	Fire
Oxygen	Not fire
Not oxygen	Not fire

As said, these are the possible scenarios for a logical formula such as $q \rightarrow p$. Nonetheless, a conjunction can be a real conditional too.

Conditional

It has also been explained that the set corresponding to the conditional in classical logic is {p & q, ¬p & q, ¬p & ¬q}, and that, for MMT, this is the usual interpretation of it. However, while this is so, Johnson-Laird and Byrne (2002) consider this last set to be just one between the ten possible interpretations of the conditional, and I am going to present now an example of conjunction that can be understood in accordance with exactly those same combinations, that is, an example of sentence with the structure $p \wedge q$ whose true logical form is $p \rightarrow q$ (for other analyses relating coordination to subordination, see, e.g., Culicover & Jackendoff, 1997). That example is this one:

You come and I leave

Undoubtedly, the possible scenarios for this sentence are the same as those of the conditional:

You come	I leave
You do not come	I leave
You do not come	I do not leave

The reason is that the sentence appears to provide that, although it is possible that the speaker leaves even if the listener does not come, if the listener comes, the speaker necessarily will leave. The only impossible scenario being hence that the listener comes and the speaker does not leave.

Otherwise, maybe it is interesting to mention here that, as in the case of disjunction, the literature of MMT on the conditional is also large and much more than Johnson-Laird and Byrne’s (2002) paper. Its adherents have authored many works about it and its possible interpretations, and some representative examples of such works can be Byrne and Johnson-Laird (2009), Khemlani et al. (2014), Quelhas, Johnson-Laird, and Juhos (2010), or Ragni, Sonntag, and Johnson-Laird (2016).

Sheffer function

Other known logical connective is Sheffer function or Sheffer stroke. Its truth-values correspond to those of a denied conjunction and, therefore, refer to the possibilities {p & ¬q, ¬p & q, ¬p & ¬q}. To find a conjunction referring to these scenarios is easy too if the fact that people often use ironic or figurative language is taken into account again. Let us consider a sentence such as this one:

Eat a lot of sweets and you will have healthy teeth

In spite of its form and its syntactic structure, its possible scenarios are:

A lot of sweets	Not healthy teeth
Not a lot of sweets	Healthy teeth
Not a lot of sweets	Not healthy teeth

And these are the correct possibilities for the sentence because what the speaker wants to state is that it is not possible to eat a lot of sweets and to have healthy teeth at the same time. All of the other scenarios are possible, including, of course, the last one, that is, not to eat a lot of sweets and not to have healthy teeth, since the former does not completely guarantees healthy teeth.

Tautology

But conjunction can also be related to the four combinations. As it is well known, in standard logic, when a formula is true in the four possible combinations, that is, when its combinations set is {p & q, p & ¬q, ¬p & q, ¬p & ¬q}, that formula is called ‘Tautology’. This same denomination is assumed by Johnson-Laird and Byrne (2002), who consider that very set to be one more interpretation of the conditional. Once again, we can base on one of their examples to present another with a conjunction. According to them, this sentence is a Tautology:

“If there are lights over there, then there may be a road” (Johnson-Laird & Byrne, 2002, p. 663).

Keeping the clauses, changing ‘if’ and ‘then’ by ‘and’, and including the verb ‘may’ in the first conjunct as well, we obtain this new sentence:

There may be lights over there and there may be a road

Obviously, all of the scenarios are possible here:

Lights	Road
Lights	Not road
Not lights	Road
Not lights	Not road

It can be thought that a reason of this can be the presence of ‘may’ in the two conjuncts. Nevertheless, it is not clearly the only reason, since that is also the case in the example of Exclusive disjunction indicated above. Thus, the crucial difference between this last interpretation and Tautology seems to be that modulation (in the example given, that is, the one of daytime and nighttime, by virtue of semantic factors) eliminates two combinations (p & q and ¬p & ¬q) in the latter.

Contradiction

Finally, although it is not as easy as in the previous interpretations, examples of conjunctions that are false under any combination of possibilities can be found as well. Therefore, it is also possible to speak about an interpretation whose set is {ø} (where ‘ø’, of course, represents the empty set).

Let us think about the following situation: a speaker is in a particular place but he/she is not paying close attention to what other people are saying. The reason is that he/she is analyzing a problem that he/she has in other place. When the other persons notice that he/she is distracted, the speaker apologizes and says:

I am here and I am in other place

If modulation, and, therefore, semantics and pragmatics, is taken into account, it can be stated that p & q is not a combination valid for this sentence, as it is impossible to be here and in other place at the same time. Likewise, ¬p & ¬q is not acceptable either, since it is also impossible neither to be here nor to be in other place at the same time. In the same way, ¬p & q is inadmissible too, as the speaker is really here, at least in a physical sense. Hence, the only remaining possible scenario would be, in principle, p & ¬q (that is, I am here and I am not in other place). The problem is that this last option does not capture the speaker's real intentions. The sentence is intended to mean that the speaker has an excuse for being absent and p & ¬q removes that excuse. In particular, it provides that the speaker is actually here and so he/she should be paying attention. Accordingly, it is difficult to accept this last combination too. The speaker, as indicated, wants to clarify that he/she is not truly here, and, for this reason, modulation can also eliminate the scenario p & ¬q. In this way, it can also be claimed that conjunction can refer to no combination of possibilities as well.

Conclusion

Therefore, it is clear that all of the sixteen interpretations that can be attributed to a connective by virtue of the truth tables of classical logic can be expressed by means of conjunctions. Everyday language includes at least sixteen kinds of conjunctions and each of those kinds seems to correspond to one of such interpretations. Thus, it can be stated that conjunction in natural language does not necessarily have the same characteristics as logical conjunction. This last conjunction refers to just the first combinations of possibilities set analyzed in this paper, and, as argued, conjunction in natural language can be related to other fifteen sets. In other words, and in short, this study shows that 'p and q' is not always $p \wedge q$, and that the latter is only one of the sixteen possible interpretations of the former.

This is a very important point, since it reveals that the general theses of MMT can be correct. Pragmatics and semantics cannot be ignored and it appears that what is essential for a sentence to be a conjunction is not that the word 'and' is in it. As seen, it is possible that the word is and the sentence is not really a conjunction. In the same way, there are obvious conjunctions that are not expressed by means of 'and'. As also explained, Johnson-Laird and Byrne (2002) called 'Ponens' to the interpretation of the conditional that matches that of conjunction in standard logic, that is, to the interpretation corresponding to the set {p & q}, and, evidently, they gave some examples in this regard as well, one of them being this:

"If my name is Alex then Viv is engaged" (Johnson-Laird & Byrne, 2002, p. 663).

There is no doubt that, if the previous sentence is said by Alex, what Alex tries to state is that the fact that Viv is engaged is as true as the fact that the speaker's name is Alex. So, the only possible scenario is p & q, and hence the conditional is actually a conjunction.

But, if this is so and pragmatics and semantics are important to that extent, it is very possible that the role played by syntax is much more secondary than thought. This last idea seems to be one of the essential assumptions of MMT, and it is probable that the findings that its proponents are obtaining with their empirical research nowadays finally show whether or not is right.

In any case, for the moment, although it is not accepted that semantics or pragmatics are more relevant than syntax in the interpretation of linguistic contents, there is at least a point that appears to be clear. That point is that the meanings of the words and the circumstances in which a sentence is said are undoubtedly decisive elements to understand the actual sense of a logical connective.

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